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Abstract

In this paper we analyze the interaction of two disciplinary mechanisms: competition and reputation. We first study a dynamic model of monopolistic competition with experienced goods (i.e., quality is observed after goods are purchased). When market power is high enough, reputation results in the equilibrium with perfect information being sustainable. If consumers' expectations satisfy a weak regularity condition, then there is a unique sequential equilibrium with quality goods being produced and the price has a mark-up which is either the full information monopolistic mark-up or, if this is not sustainable (e.g., when goods are very close substitutes), the rate of time preference, that acts as a reputation constraint. A variation of the model allows us to study the private provision of currencies. In particular, we inquire whether Bertrand competition between profit maximizing currency issuers would drive inflation rates to the efficient outcome, as suggested prominently by Hayek. We show that, unless firms can commit to future actions, the efficient outcome is never attained. Without full commitment, equilibria with deflation -as implied by the Friedman rule- can not be sustained, however, if currencies are close substitutes (and beliefs regular) the equilibrium inflation rate is zero.

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1 Introduction

Can currency be efficiently provided by competitive markets? As we show in this paper, such a *macro* question has a very similar *micro* formulation: can experienced goods (i.e., goods where quality is observed after being purchased) of high quality be provided by competitive markets? In this paper, we study a dynamic monopolistic equilibrium model to provide an answer to the *micro* question. A variation of the model allow us to answer the original *macro* question. Both questions address the more general theme of how the reputation and the competitive mechanisms interact. This is the central theme of the paper.

In monetary theory, the standard *laissez-faire* view –as, for example, has been expressed by Hayek– is based on a "Bertrand competition" argument, according to which competition will drive the price (return) of money to its marginal cost. Given the real rate of interest, and abstracting from legal restrictions or acceptability issues, the demand for a given currency depends on its real return. Therefore, a competitive issuer will try to undercut competitors with a lower inflation rate. If the marginal cost of producing currency is zero, this process of competition should make the inflation rate equal to the symmetric of the real rate of interest. That is, an "optimal monetary policy" of zero nominal interest rates should be the result of competition.

There is, however, one major flaw in this "Bertrand competition" argument, when applied to fiat money. When suppliers of currency cannot commit to their future actions, then competition may lose its bite. The reason being that, while currencies compete on their promised rates of return, once agents hold a particular currency there may be an incentive for the issuer to inflate the price of goods in terms of this currency, reducing, in this way, the outstanding liabilities. This problem, notice, is at the root of the time inconsistency problem in monetary theory. Current currency portfolios have been pre-specified, while there is full flexibility to choose tomorrow's portfolios. Currencies compete for tomorrow portfolios. Thus, Bertrand competition drives *promised* rates of return to the efficient level. But those promises might not be credible.

That is, the currency supplier has an incentive to default on the promise made while competing with other currencies. In other words, when the choices are sequential, currencies are no longer perfect substitutes; in a sense, they are not substitutes at all!

In the industrial organization theory, competition in experienced good markets has similar properties. "Bertrand competition" can only affect market prices, but not the qualities which are observed *ex-post*. Firms have an incentive to "fly-by-night" providing low quality products. This problem, notice, is at the root of the "market for lemons" problem in industrial organization theory. In summary, in a monopolistic competition context, even if goods are close substitutes, competition does not discipline firms.

In a dynamic economy, however, firms are concerned for their future market position and such reputation mechanism may be enough to discipline firms to provide high quality goods. Similarly, the reputation mechanism may resolve the time inconsistency problem in the supply of money: concern for the future circulation of money may disincentive currency issuers to create unexpected inflations. Nevertheless, reputational concerns exist as long as firms, monopolistic firms in one contest and currency suppliers in the other, expect high enough future profits as to refrain from capturing the short-term profits. Therefore, competition, driving down profits, may enhance efficiency, but may also destroy the disciplinary properties of the reputation mechanism. The analysis of this trade-off is the central contribution of this paper.

In studying monopolistic competition we first consider the case of perfect observability where, as the well understood theory predicts, equilibrium is uniquely determined by the degree of substitution and, as goods become closer substitutes, equilibria become more efficient, achieving Pareto efficiency in the limiting case of perfect substitution. However, when quality is only observed with a lag the mark-up must be bounded away from zero as to guarantee enough future profits for reputation to play a role. The rate of time preferences -as indicator of the observability lag- defines the lower bound on mark-ups. More precisely, we first show that any price covering such mark-up could be sustained as a symmetric

stationary *sequential equilibrium* (and as a *sustainable equilibrium).* This also means that competition does not play any role since, for example, any arbitrary large price can be an equilibrium price, sustained by beliefs that price deviations, from such a high price, signal low quality. That is, arbitrariness of beliefs results in arbitrary price equilibria.

We then constraint beliefs to satisfy two weak regularity conditions which are consistent with most learning procedures. In other words, we constraint rational expectations equilibria to be *learnable* in the sense of being supported by beliefs that satisfy minimum regularity conditions, such as a weak forms of continuity and monotonicity. We introduce this way, the concept of *regular sequential equilibrium*, which is distinct from other existing refinements and can be of interest in other applications. In contrast with sequential equilibria, there is a unique stationary regular sequential equilibrium, in which competition plays a crucial role. In particular, the mark-up is either the mark-up of the monopolistic competition with perfect observability or, if goods are too close substitutes as such a price not being sustainable, the -lower bound- rate of time preferences.

Our model of currency competition is one where goods are supplied in perfectly competitive markets, but consumers, who would like to consume all goods, must pay for each good with a specific currency, that is, they face multiple (a continuum) of cash-in-advance constraints (as in Woodford 1990). Each currency is supplied by a profit maximizer provider, therefore currency competition place the role of monopolistic competition in the original model. The parallel with the case of perfect observability is the case with full commitment. Currency competition achieves the efficient (Friedman rule) monetary equilibrium if currencies become perfect substitute, as Hayek had envisioned. However, the parallel with the case of unobservable quality is the case without full commitment. Both cases share the property that competition is on announced prices (interest rates), while consumers base their demands on expectations on whether such announcements will be realized. The equivalent of the lower bound on mark-ups is a non-negative inflation condition, which must be guaranteed for reputation to play a role. As

in the monopolistic competition model, there is a large set of sequential equilibria, which shrinks to a unique equilibrium when beliefs satisfy regularity conditions. In particular, the unique stationary *regular sequential equilibrium* is characterized by an inflation that it is either zero or the inflation of the full commitment equilibrium, if such inflation rate is positive. It follows, for example, that the efficient (Friedman rule) monetary equilibrium can not be sustained as Hayek had envisioned. It also follows, however, that, with rent maximizing currency providers, competition enhances efficiency, contradicting some of the arguments of those advocating that "money is a natural monopoly".

Although we exploit the similarities between experienced goods and monies, such analogy should not be pushed too far. For example, there is a "coordination problem" associated with the "acceptance of fiat money," which is more similar to "coordination problems" associated with some products, such as "fashion goods". In contrast with the industrial organization literature, in monetary theory often the argument for "money as a natural monopoly" has been made based on the existence of such a "coordination problem". In this paper we will not address such endogeneity problems. We will, however, take into account another differential feature of fiat money: the fact that if people do not expect that a currency will be held in the future, then their current demand for such a currency is zero. In this regard money is more like a non-perishable durable good that requires good maintenance for life. That is, it is enough that you do not expect to find service (of quality) in the future for you not to buy -say, a car of a certain maker- even if current quality (both of the car and the service) is the appropriate one.

Our work is related to different strands of literature. With respect to the industrial organization literature on experience goods, our work is closely related to Shapiro (1983)[1]. He considers a similar model of monopolistic competition in which consumers' expectations regarding quality follow an ad-hoc exogenous process. He does not study the trade-offs between "competition and reputation". In contrast, we consider rational expectations about quality and, as we have said, our central theme is the

[1]See also Tirole (1988) for an introductory account.

study these trade-offs.

The issue of currency competition has been the subject of an extensive academic debate. This debate has seen many supporters of free competition making an exception when it comes to money (as Friedman, 1960), while advocates of free currency competition (notably, Hayek1974 and 1978, and Rockoff, 1975) have been somewhat isolated . Although, the recent reappraisal of the self-regulating properties of free banking[2], has raised new interest on the study of currency competition. Woodford (1990) studies currency competition in a model similar to the one studied here. He is interested in studying the stability of exchange rates and does not consider reputational issues.

The problem of the *time-inconsistency of monetary* policies has been extensively studied (see, for example, Chang (1998), Chari & Kehoe (1990), Ireland (1994), Stokey, 1991), but with the partial exception of Taub (1985), the "currency competition" argument has not been considered[3]. Taub (1985) studies two commitment regimes: one with full commitment with non-stationary ("time-inconsistent") policies, and another in which polices are constrained to be "time-consistent" (stationary). He shows that in the commitment case, the Friedman rule emerges as the competitive outcome, while in the "time-consistent" case the outcome is inefficient and, as a result, he argues in favor of the "natural monopoly" argument. While we have the same result when there is full commitment, our analysis of the "non-commitment case" differs substantially, showing how, and when, competition enhances efficiency. Finally, the paper which is closest to this one, is our, Marimon, Nicolini and Teles (1998), companion paper on the effects on (the unique) monetary policy of competition from electronic money, and other currency substitutes.

[2]See, for example, Calomiris and Kahn (1996), Dowd (1992), King (1983), Rolnick and Weber (1983), Selgin (1987), Selgin & White (1987), Vaubel (1985), and, more generally, White (1993) . See also Schuler (1992), for an account of historical episodes of free banking, and Hayek (1974,1978), Dowd (1992) and White (1993) for a broad perspective on the literature on free banking.

[3]A shortcoming that has not gone unnoticed (see, for example, Hellwig (1985)).

2 A model of monopolistic competition with experienced goods

Our model is a version of the model of monopolistic competition of Dixit and Stiglitz (1977) with experienced goods. Consider an economy with a large number of identical consumers that gain utility form services and leisure. The utility functions are

$$\sum_{t=0}^{\infty} \beta^t \left[U(y_t) - \alpha n_t \right], \tag{1}$$

where U is increasing and concave and, without loss of generality, $U(0) = 0$, α is a positive constant, n_t is work effort and q_t is an index of services

$$y_t = \left[\int_0^1 \left(y(i)_t q(i)_t \right)^{1/\mu} di \right]^{\mu},$$

with $\mu \geq 1$. $y(i)_t$ is the consumption of good $i \in [0,1]$. Each of the goods can be provided with variable quality, $q(i)_t = 0$ or 1.

Time must be devoted to the production of services, according to the linear technology

$$y(i)_t q(i)_t = n(i)_t,$$

We assume that there is a single monopolist that produces each good. Total effort per capita is

$$n_t = \int_0^1 n(i)_t di.$$

Producers have, at any time, the option of producing "fake" units of the consumption good that are costless to produce. A key assumption for the characterization of the equilibria is whether consumers can distinguish the good quality goods from the bad quality ones before they buy them. We proceed to characterize the equilibrium when the services obtained with the consumption of the goods are observed before they are purchased.

2.1 Monopolistic competition with perfect observability

If the quality of the good is public information, there exists a unique equilibrium in this model economy with monopolistically competitive firms. If the quality of the good is public information, there exists a unique equilibrium in this model economy with monopolistically competitive firms.

In each period t, the consumers choose the number of units of each good i to purchase, $y(i)_t$, as well as work effort, n_t, in order to maximize utility, (1), subject to

$$\sum_{t=0}^{\infty} Q_t \left[\int_0^1 \left(p(i)_t y(i)_t - \Pi(i)_t \right) di - n_t \right] \leq 0,$$

where $\Pi(i)_t$ are the per-capita profits of firm i, $p(i)_t$ is the price of goods in units of labor time, the numeraire, and Q_t is the price of labor at time t, in units of labor at time zero. The demand functions for goods will be given by

$$U'(y_t) y_t^{\frac{\mu-1}{\mu}} \left(y(i)_t q(i)_t \right)^{\frac{1-\mu}{\mu}} q(i)_t = \alpha p(i)_t, \tag{2}$$

for all i and t. When $q(i)_t = 0$, then $y(i)_t = 0$.

When instead $q(i)_t = 1$, the price of the composite good y_t is

$$p_t \equiv \left[\int_0^1 p(i)_t^{1/1-\mu} di \right]^{1-\mu} = \frac{U'(q_t)}{\alpha}$$

The demand functions for services of each of the goods, (2), can then be written as

$$y(i)_t = y_t \left[\frac{p(i)_t}{p_t} \right]^{\frac{\mu}{1-\mu}} \tag{3}$$

The monopolist of product i chooses the quality and the price to maximize profits

$$\sum_{t=0}^{\infty} \beta^t \left(p(i)_t y(i)_t - q(i)_t y(i)_t \right). \tag{4}$$

Since with $q(i)_t = 0$, $y(i)_t = 0$, and profits will be zero, then the firms will provide the high quality, $q(i)_t = 1$. They choose the prices to maximize profits (4) subject to the demand functions (3). The problem is static. As the demand function has constant price elasticity, the optimal price per unit of service of each good will be

$$p(i)_t = \mu. \tag{5}$$

The market clearing condition

$$\int_0^1 q(i)_t y(i)_t di = n_t$$

must hold in equilibrium.

The unique equilibrium will be characterized by a price which will be constant over time and across goods

$$\overline{p} = \mu, \tag{6}$$

as equation (5) shows. Therefore, the quantity of services of the goods, $y_t = \overline{y}$, will be constant and will satisfy the following condition

$$U'(\overline{y}) = \alpha\mu \tag{7}$$

The value of the parameter μ determines the substitutability of the goods. The closer is μ to one, the higher is the degree of substitutability. Note that when μ is in fact one, the mark-up goes to zero and the equilibrium is a perfectly competitive one. On the other hand, as μ gets larger, so do the mark-ups. Note that we are not allowing for free entry, so profits will indeed be positive except in the limiting case in which $\mu = 1$.

Thus, there exists a unique equilibrium that is closer to the efficient outcome, the closer is the parameter μ to one. Indeed only when $\mu = 1$, the marginal rate of substitution equals the marginal rate of transformation. The increased substitutability between goods increases competition and increased competition implies an outcome closer to the efficient one. This models thus illustrates in a very clear way the nice properties of competition.

2.2 Monopolistic competition with unobservable quality

We now assume that, as with many durable goods, consumers can observe the quality of the good -or service- only after purchasing it. This feature modifies the model above in very important ways. In particular, note that each firm now faces a "time inconsistency problem". As is clear from the expression for profits (4), in each period t, once the consumers have paid the price of the good, $p(i)_t$, under the expectation that the good is of high quality, $q(i)_t = 1$, it is optimal to provide no services, $q(i)_t = 0$, and save the costs of production[4]. Of course, the firms will refrain from doing so, if this action can affect future demand, since after observing low quality the consumers might choose $y(i)_{t+j} = 0$, $j \geq 1$. In this section, we develop a model of reputation to analyze this problem.

The first thing to notice is that a firm will never announce that it is producing low quality services, therefore, a price announcement can be understood as a price announcement of a high quality service. Let $\widetilde{p(i)_t}$ denote the announced price by firm i in period t for a good or service, supposedly of high quality. Then, the realized price is $p(i)_t = \widetilde{p(i)_t}/q(i)_t$. In other words the announced price is the realized price if, and only if, the firm chooses $q(i)_t = 1$. Let h_t be the information available to a firm at the moment of making their period t decisions. That is, $h_0 = \{\emptyset\}$ and, for $t > 0$, $h_t = \{h_{t-1}, p(i)_{t-1}, \text{ for all } i\}$. More formally, a strategy for firm i, is a $\sigma_i^f = \left\{\sigma_{i,t}^f\right\}$, where, $\sigma_{i,t}^f(h_t) = \left(\widetilde{p(i)_t}, q(i)_t\right) \in R_+ \times \{0,1\}$. A consumer simply decides how much to work and to purchase of every service at current announced prices, given his available information, which in period t is $\left(h_t, \widetilde{p(i)_t}, \text{ for all } i\right)$. In other words, a strategy for a -representative- consumer is a $\sigma^c = \{\sigma_t^c\}$, where, $\sigma_t^c(h_t, p_t) = \{n_t, y(i)_t, \text{for all } i\}$. Consumers' decisions, however, are based on their current beliefs. Let $\upsilon_t^{c,i}(h_t, \widetilde{p(i)_t})$ denote the belief that, given history h_t,and price announcement $\widetilde{p(i)_t}$ the realized price is the announced one,

[4]This feature has not been unnoticed in the Industrial Organization literature (see Shapiro 1983). However, to the best of our knowledge, the problem has not been analyzed in the context of fully rational agents.

i.e., $p(i)_t = \widetilde{p(i)}_t$. In other words, the consumer's attaches a probability $\upsilon_t^{c,i}(h_t, \widetilde{p(i)}_t)$ to firm i producing with quality $q(i)_t = 1$ if it announces a price $\widetilde{p(i)}_t$. Notice that we implicitly assume that beliefs about firm i do not depend on other firms' announcements.

As in Kreps and Wilson (1982)' *Sequential Equilibrium* and as in *Perfect (Extended) Bayesian Equilibrium*[5], consumers' beliefs must be consistent with firms's strategies. More precisely, we may allow firms to make random choices and denote by $\sigma_{i,t}^{f,1}(h_t)[\ \widetilde{p(i)}_t] = \Pr\{\ \sigma_{i,t}^{f,1}(h_t) = \widetilde{p(i)}_t\}$ and, similarly, $\sigma_{i,t}^{f}(h_t)[\ \widetilde{p(i)}_t, p(i)_t] = \Pr\{\sigma_{i,t}^{f}(h_t) = \left(\widetilde{p(i)}_t, 1\right)\}$. An *assessments* $\left((\sigma^c, \upsilon^c), (\sigma_i^f)\right)$*is said to be consistent* if for every (t, h_t) and $\widetilde{p(i)}_t$ with $\sigma_{i,t}^{f,1}(h_t)[\ \widetilde{p(i)}_t] > 0$

$$\upsilon_t^{c,i}(h_t, \widetilde{p(i)}_t) = \frac{\sigma_{i,t}^{f}(h_t)[\widetilde{p(i)}_t, p(i)_t]}{\sigma_{i,t}^{f,1}(h_t)[\widetilde{p(i)}_t]}$$

We can now define a "*Sequential Monopolistic Competition Equilibrium*" (SMCE) as an *assessment* $\left((\sigma^c, \upsilon^c), (\sigma_i^f)\right)$ which is *consistent* and, such that, for every (t, h_t)

i. $\sigma_t^c(h_t, \widetilde{p(i)}_t)$ solves the problem of the consumer given his beliefs $\upsilon_t^{c,i}(h_t, \widetilde{p(i)}_t)$.

ii. $\sigma_{i,t}^f(h_t^f)$ solves the problem of firm i.

iii. The market clearing condition $\int_0^1 y(i)_t q(i)_t di = n_t$ is satisfied.

"Sequential Monopolistic Competition Equilibrium" (SMCE) provides a natural framework to study the interactions between competition and reputation[6]. On the one hand, as long as μ is strictly larger than

[5]In our imperfect information context, our consistency condition satisfies the limiting consistency condition of *sequential equilibrium*, as well as the consistency condition on relative beliefs of *perfect extended Bayesian equilibrium* (see, for example, Fudenberg and Tirole, 1991, Ch. 8).

[6]Notice that conditions (1) and (2) correspond to Kreps and Wilson (1982)' *Sequential Rationality*. We could have imposed, instead, that sequential rationality be

one, the economy exhibits monopolistic power, and as μ gets close to one, the competition between firms is increased. On the other hand, in making quality decisions, firms care about their reputation since quality provision has strategic implications.

In what follows, we restrict attention to symmetric equilibria in the sense that all firms behave the same way so expectations about quality are the same for every good. We also make use of the fact that, given beliefs regarding quality, $\upsilon_t^{c,i}(h_t, \widetilde{p(i)}_t)$, consumers' optimal decisions are, in fact, their demands. Given $\left(h_t, \widetilde{p(i)}_t, \overline{p}\right)$, where $\overline{p} \equiv \left[\int \widetilde{p(i')}_t^{1/(1-\mu)} di'\right]^{1-\mu}$, and $\eta \equiv \upsilon_t^{c,i}(h_t, \widetilde{p(i)}_t)$ the consumer's demand is given by

$$U'(y_{\overline{p}}) y_{\overline{p}}^{\frac{\mu-1}{\mu}} \left(y(i)_t\right)^{\frac{1-\mu}{\mu}} = \alpha \frac{\widetilde{p(i)}_t}{\eta}$$

that is,

$$\begin{aligned} y(i)_t &= \sigma_t^{c,i}(h_t) = d(\widetilde{p(i)}_t, \overline{p}; \eta) \\ &\equiv y_{\overline{p}} \left[\frac{\overline{p}}{\widetilde{p(i)}_t}\right]^{\frac{\mu}{\mu-1}} (\eta)^{\frac{\mu}{\mu-1}} \end{aligned} \tag{8}$$

In order to stress the pervasive effects of assuming that the quality is only observed with a lag, let us consider an equilibrium where strategies do not depend on histories. If current actions of the firms do not affect the consumers' expectations about future quality, then, no matter what the price is, it is a dominant strategy for the firms to choose to provide the low quality, $q(i)_t = 0$, to save on production costs. If the firm produces low quality, then for any price announcement $\widetilde{p(i)}_t$ if $\upsilon_t^{c,i}(h_t, \widetilde{p(i)}_t) = 0$, consumer's expectations are fulfilled[7]. The resulting payoffs are zero and

satisfied, not in every possible history, but only along histories which -possibly, with zero probability- could only ocurr along equilibrium paths. Such weaker requirement corresponds to the cocept of *Sustainable Equilibrium* of Chari and Kehoe (1990). As we will see, there is a fairly large set of *Sequential Equilibria* which are, of course, *Sustainable Equilibria.*

[7]Notice that the corresponding assesment is consistent since it is enough to consider a sequence of assesments $\left((\sigma^c, \upsilon^c)_n, (\sigma_i^f)_n\right)$ with $\sigma_{i,t}^f(h_t^f) = \left\{\widetilde{p(i)}_t, \varepsilon_n\right\}$, $\upsilon_t^{c,i}(h_t^c) = \varepsilon_n$,

since there are no other forms of punishment it follows that this is the worst SMCE. More formally,

Proposition 1 *Consumers' beliefs* $\upsilon_t^{c,i}(h_t^c) = 0$, *with their corresponding strategies* $\sigma_t^c(h_t^c) = (0,0)$, *for all* $h_t^c \in H_t^c$, *and firms' strategy* $\sigma_{i,t}^f(h_t^f) = \left(\widetilde{p(i)}_t, 0\right)$, *for any* $\widetilde{p(i)}_t \geq 0$, *for all* i *and* $h_t^f \in H_t^f$ *define a (low quality) SMCE. Furthermore, there is no SMCE with lower payoffs for consumers and firms.*

Incidentally, note that this is the unique SMCE (payoff) in which strategies do not depend on histories. In this case no reputation considerations arise[8]. Note also that this would be the unique outcome if firms where anonymous players not accountable for their past quality decisions.

The next step is to determine under what conditions the equilibrium with perfect observability[9], described in the previous section, is a SMCE. In order to check this, we follow the literature and consider reversion to the worst SMCE strategies.

Consider the outcome

$$q(i)_t = 1, \widetilde{p(i)}_t = \overline{p},\ y(i)_t = y_{\overline{p}}, n_{\overline{p}} = y_{\overline{p}}$$

where $U'(y_{\overline{p}}) = \alpha\overline{p}$. We want to find conditions under which this outcome is supported by the following "revert to low quality" strategies and beliefs:

$$\begin{aligned} \sigma_{i,0}^f &= (\overline{p}, 1)\,, \\ \sigma_t^f(h_t) &= (\overline{p}, 1)\,, \text{ if } q(i)_n = 1 \text{ for } 0 \leq n < t, \\ &= (\overline{p}, 0)\,, \text{ otherwise.} \end{aligned}$$

$\sigma_t^c(h_t^c) = d(\widetilde{p(i)}_t, \overline{p}; \varepsilon_n)$ and having $\varepsilon_n \searrow 0$. However, if we required that firms maximize profits along the limiting sequence of assesments, then $\widetilde{p(i)}_t \searrow 0$ as $\varepsilon_n \searrow 0$. That is, only a zero price is can be a *Perfect Sequential Competitive Equilibrium* with low quality. See, for example, Fudenberg and Tirole (1991) for a discussion of trembling-hand perfection and related concepts.

[8]In the repeated game literature this outcome is called the outcome of the static game.

[9]In the time consistency literature, this is the commitment outcome. In our model, observability is the commitment technology.

$$
\begin{aligned}
\upsilon_0^{c,i}(h_0,\ \widetilde{p(i)_0}) &= 1 \text{ and } \sigma_0^{c,0}(h_0^c) = n_{\overline{p}} \text{ if } \widetilde{p(i)_0} = \overline{p}, \\
\upsilon_0^{c,i}(h_0,\ \widetilde{p(i)_0}) &= 0 \text{ and } \sigma_0^{c,0}(h_0^c) = 0, \text{ otherwise} \\
\upsilon_t^{c,i}(h_t,\ \widetilde{p(i)_t}) &= 1, \text{and } \sigma_0^{c,0}(h_0^c) = n_{\overline{p}} \text{ if } q(i)_n = 1, 0 \leq n < t \text{ and } \widetilde{p(i)_t} = \overline{p} \\
\upsilon_t^{c,i}(h_t^c,\ \widetilde{p(i)_t}) &= 0, \text{and } \sigma_0^{c,0}(h_0^c) = 0, \text{ otherwise.}
\end{aligned}
$$

given that consumption strategies are given by (8), it follows that

$$
\begin{aligned}
\sigma_0^{c,i}(h_0,\ \widetilde{p(i)_0}) &= y_{\overline{p}} \text{ if } \widetilde{p(i)_0} = \overline{p}, \sigma_0^{c,i}(h_0,\ \widetilde{p(i)_0}) = 0 \text{ otherwise} \\
\sigma_t^{c,i}(h_t^c,\ \widetilde{p(i)_t}) &= y_{\overline{p}}, \text{ if } q(i)_j = 1, 0 \leq n < t \text{ and } \widetilde{p(i)_t} = \overline{p} \\
\sigma_t^{c,i}(h_t^c,\ \widetilde{p(i)_t}) &= 0, \text{otherwise.}
\end{aligned}
$$

Consider first the monopolistic competition outcome with perfect observability; that is, $\overline{p} = \mu$ and $y_{\overline{p}} = \overline{y}$. Notice that, if it is sequentially rational for firms to produce high quality with probability one, then the above strategies correspond to sequentially rational choices, since they where optimal choices with perfect observability.

If the firm does indeed deliver the high quality good, then the profits, each period, will be given by $\Pi(i) = (\mu - 1)\overline{y}$ and, therefore, the present value profits, after high quality is observed all previous periods and the current price is μ, are given by $V(\mu, 1) = (\mu - 1)\overline{y}/(1 - \beta)$. On the other hand, if the firm deviates and delivers the low quality good, the current profits will be $\mu\overline{y}$ and the present value profits, after quality zero (i.e., $p(i)_{t-1} = \infty$) is observed the last period (or any previous period), are $V(\mu, 0) = 0$. Thus, the firm chooses not to deviate and produce high quality (i.e., $\widetilde{q}(i)_t = 1$ is sequentially rational) if

$$
\begin{aligned}
& (\mu - 1)\overline{y} + \beta V(\mu, 1) \\
\geq\ & \mu\overline{y} + \beta V(\mu, 0)
\end{aligned}
$$

that is, if

$$
\beta V(\mu, 1) = \frac{\beta}{(1 - \beta)}(\mu - 1)\overline{y} \geq \overline{y} \tag{9}
$$

Let $\beta = 1/(1 + \rho)$, then the firm will choose not to deviate when

$$
\mu \geq 1 + \rho.
$$

Notice, furthermore, that the resulting assessment is consistent. Thus, we have shown the following proposition

Proposition 2 *If the market power is high enough, so that the mark-up is higher than the discount rate, then the perfect information equilibrium is a SMCE.*

The intuition of the last proposition is clear. Given that the firm has the option of making a short run profit by selling low quality goods, the equilibrium mark-up must be high enough for the firm not to choose to do it. As the equilibrium profits are accrued over time, the discount rate matters.

This is the intuition of the Industrial Organization literature on unobservable quality, and the first quotations go back to Adam Smith. If by reducing the quality the firm can make short run profits, a reputation argument can explain why firms decide not to do so. As we have just seen, reputation is valuable when firms make positive profits in equilibrium. But, as competition gets tighter, i.e., in our model μ gets arbitrarily close to 1, monopolistic rents disappear and the equilibrium with perfect observability may not be sustainable through reputation if discount rate is high enough. Nevertheless, the time period can be seen as the time that it takes for consumers to observe the quality of the goods. The shorter is the information lag, the smaller is the discount rate, and the easier to sustain the equilibrium with perfect observability.

So far, we have only shown under which conditions is the perfect information equilibrium a SCE. However, it should be noticed that the above argument applies to any outcome defined by $\overline{p} = \mu'$ and $y_{\overline{p}} = y'$, as long as $\mu' \geq 1 + \rho$. In this case, the choices are sequentially rational since consumers satisfy their demands at $\widetilde{p(i)}_t = p(i)_t = \mu'$, firms make non-negative profits and any deviation is punished. In particular, a price deviation is instantaneously punished by triggering the beliefs that the firm is producing low quality. It follows that a price deviation is dominated by choosing to announce $\widetilde{p(i)}_t = \mu'$ and delivering high quality. In summary,

Proposition 3 *For any $\mu' \geq 1+\rho$ there exists a SMCE where the price per unit of service is μ' and firms always produce the high quality good.*

Note that a particular feature of the equilibria discussed above is that all firms must be making positive profits in equilibrium, so that the Pareto efficient solution is never attained, as long as $\rho > 0$.

This analysis shows how high quality can be maintained through reputation, but there is no role for competition. In fact, as consumer expectations depend on price and quality history, there are no dimensions along which the firms can actually compete. There is a sense in which consumers expectations alone determine the whole set of restrictions that the firms face, so the actions of a single firm end up by being irrelevant for the others. The problem, however, is more with the definition of SMCE than with competition. We next discuss this problem, showing that a more "sensible" definition of equilibrium results in competition playing a role.

2.2.1 Regular beliefs

The first thing to notice is that in a SMCE consumers' beliefs may be fairly unreasonable. For example, conditional on past history, beliefs can be highly discontinuous in current prices, a consumer may have had his expectations of high quality always fulfilled in the past and, yet, a price announcement trigger a complete distrust. In fact, nothing precludes that beliefs could be such that observing high quality were to lead to down grading consumers' expectations of high quality. It seems more natural (more consistent with any reasonable learning process) to assume that beliefs satisfy some minimal continuity and monotonicity properties. We impose a regularity condition on beliefs that takes into account these intuitive ideas[10].

[10] To our knowledge, our regularity condition is new and distinct from existing *refinements.* Nevertheless, we are not the first ones to introduce monotonicity conditions on beliefs. For example, Kreps and Wilson (1982) considered similar restrictions.

Definition 1 *A consumer has* $\varepsilon-$ positive beliefs *if there exist a* $\varepsilon > 0$ *such that, for all* $i, h_t, \widetilde{p(i)}$, $\upsilon_t^{c,i}(h_t, \widetilde{p(i)}) \geq \varepsilon$ *if there is no* $n = 0, ..., t-1$ *with* $p(i)_n \neq \widetilde{p(i)}_n$.

Definition 2 *A consumer has* weakly monotone beliefs *if* $\upsilon_{t+1}^{c,i}(h_t, \widetilde{p(i)}, \widetilde{p(i)}/1, \widehat{p(i)}) \geq \upsilon_t^{c,i}(h_t, \widetilde{p(i)})$, *for all* i, h_t, $\widetilde{p(i)}$ *and* $\widehat{p(i)} \geq \widetilde{p(i)}$

Definition 3 *An* $\varepsilon-$ Regular SMCE *is a SMCE where agents' beliefs are* ε*-positive and weakly monotone.*

Definition 4 *A* Regular Monopolistic Competition Equilibrium *(RMCE) is an assessment* $\left((\sigma^c, \upsilon^c), (\sigma_i^f)\right)$, *such that there is a sequence of* ε_n*-Regular SMCE satisfying* $\left((\sigma^c, \upsilon^c)_n, (\sigma_i^f)_n\right) \to \left((\sigma^c, \upsilon^c), (\sigma_i^f)\right)$ *as* $\varepsilon_n \searrow 0$.

That is, *ε-positive beliefs* incorporate an element of trust. Consumers' beliefs must assign at least ε probability of delivering high quality, as long as firms have always fulfilled consumer's expectations. As with perfection arguments, we then consider assessments that can be limit of equilibrium assessments with such an element of trust. *Weakly monotone beliefs* have the property that believing the announced price will be realized, i.e., $p(i) = \widetilde{p(i)}/1$, when $\widetilde{p(i)}$ is being announced should be reinforced after such expectation has been realized in the past. As it can be seen, our *regularity* conditions are fairly weak and reasonable. They are typically satisfied when beliefs evolve (i.e., are updated) according to some learning procedure. For example, *Bayesian updating* will satisfy *weak monotonicity* if consumers consider that they are observing a stationary path; as in fact they are in a stationary RMCE. Similarly, *ε-positiveness* is satisfied when the learning process starts with not degenerated initial beliefs and, again, satisfies minimal monotonicity conditions guaranteeing that a forecast of low quality with probability one will not be made after only having observed high quality.

To see the role that such regularity conditions can play, consider that firms follow a strategy with stationary prices and quality $(\overline{p}, 1)$, part of a SMCE, and firm i considers to deviate in period t and announce

$\widetilde{p(i)} \geq 1$ from then on an maintain high quality. Given that along the path price announcements are always realized $\upsilon_t^{c,i}(h_t, \widetilde{p(i)}) \geq \varepsilon_n$ in a ε_n-RSCE. In this case, firm i will deliver high quality in period t if the following incentive condition, equivalent to (9), is satisfied

$$\begin{aligned} & \beta \sum_{n=0}^{\infty} \beta^n (\widetilde{p(i)} - 1) y_{\overline{p}} \left(\frac{\overline{p}}{\widetilde{p(i)}} \right)^{\frac{\mu}{\mu-1}} \left(\upsilon_{t+n+1}^{c,i}(h_t, \widetilde{p(i)}, (\widetilde{p(i)}/1, \widetilde{p(i)})_{t+1}^{t+n+1}) \right)^{\frac{\mu}{\mu-1}} \\ \geq \quad & y_{\overline{p}} \left(\frac{\overline{p}}{\widetilde{p(i)}} \right)^{\frac{\mu}{\mu-1}} \left(\upsilon_t^{c,i}(h_t, \widetilde{p(i)}) \right)^{\frac{\mu}{\mu-1}} \end{aligned}$$

where $(\widetilde{p(i)}/1, \widetilde{p(i)})_{t+1}^{t+n+1}$ denotes the sequence of observed prices $p(i)_m = \widetilde{p(i)}/1$, $m = t, ..., t+n$ and announcements $\widetilde{p(i)}_s = \widetilde{p(i)}$, $s = t+1, ..., t+n+1$. That is,

$$\beta(\widetilde{p(i)} - 1) \sum_{n=0}^{\infty} \beta^n \left(\upsilon_{t+n+1}^{c,i}(h_t, \widetilde{p(i)}, (\widetilde{p(i)}/1, \widetilde{p(i)})_{t+1}^{t+n+1}) \right)^{\frac{\mu}{\mu-1}} \geq \left(\upsilon_t^{c,i}(h_t, \widetilde{p(i)}) \right)^{\frac{\mu}{\mu-1}} \tag{10}$$

However, if beliefs satisfy the *weak monotonicity* condition (10) reduces to

$$\rho^{-1}(\widetilde{p(i)} - 1) \geq 1$$

or

$$\widetilde{p(i)} \geq 1 + \rho$$

In other words, firm i will maintain high quality as long as the announced price satisfies the mark up reputational condition. It follows that a profit maximizing firm will deviate to

$$p(i) = \max\{\mu, 1 + \rho\} \tag{11}$$

Therefore, since for any ε_n-RSCE (11) is satisfied, it must also be satisfied in a RSCE. Furthermore, the previous argument also shows that the worst SCE is not a RSCE since, given that beliefs are not degenerate in period zero, firms always prefer to start offering high quality. Nevertheless, in the zero probability event that low quality is observed, the worst SCE path is part of the RSCE since after low quality has been

observed our regularity conditions do not place any restriction on beliefs. We can now state the main proposition that relates competition and reputation.

Proposition 4 *There is a unique stationary* Regular Monopolistic Competition Equilibrium *(RMCE), which is characterized by the production of high quality services being sold at a per unit price of* $\mu' = \max\{\mu, 1+\rho\}$.

Notice that, by making very weak assumptions on beliefs, we have obtained very strong results. With competition and reputation, there is a unique stationary equilibrium where the mark up is the maximum between the mark up of the competitive equilibrium with perfect observability and the interest rate. As we noticed before, it follows that the RMCE can not be efficient (even if $\mu \searrow 1$) as long as $\rho > 0$.

3 A model of currency competition

In this section we modify the model described above to introduce competition between profit maximizing currency issuers. Our aim is to show how competition and reputation interact in the private provision of money and, in particular, if it can be an efficient monetary arrangement.

As in the monopolistic model, there is a continuum of goods or services and consumers' preferences are given by $\sum_{t=0}^{\infty}\beta^t\left[U(y_t)-\alpha n_t\right]$, where as U satisfies the same monotonicity and concavity assumptions and $y_t = \left[\int_0^1 y(i)_t^{1/\mu}di\right]^{\mu}$ with $\mu \geq 1$. In contrast with the monopolistic model, we allow for free entry in the production of each of the goods. That is, product markets are competitive and, since the technology is linear in labor, firms will make zero profits in equilibrium. It follows that all goods or services have a per unit price of *one* and that the real wage will be constant and equal to one.

The central characteristic of our model of currency competition is the existence of currency issuers, each one having the right (monopoly) to

issue its own distinct currency, and each specific currency being needed to purchase a corresponding specific good. More precisely, we impose a money-specific cash-in-advance constraint on each good. To simplify, we will denote by currency i the currency needed to purchase good i. It follows that as μ approaches one currency substitution increases. In particular, currencies are perfectly substitutable in the limiting case of $\mu = 1$. We assume that currency issuers take into account the demands for their currencies, taking real interest rates as given. We also assume that they are owned by households.

Thus, the representative consumer maximizes utility subject to the following budget constraint

$$b_{t+1} + \int_0^1 \left[\frac{M(i)_{t+1}}{P(i)_t} + y(i)_t \right] di \leq n_t + b_t(1 + r_t) + \int_0^1 \frac{M(i)_t}{P(i)_t} di + \Pi_t$$

where $P(i)_t$ is the price of good i in units of money i, $\Pi(i)_t$ are the current profits of the provider of currency $i, \Pi_t = \int_0^1 \Pi(i)_t di$, b_{t+1} are real bonds measured in units of the composite good, and r_t is the real interest rate.

The cash-in-advance constraints are

$$P(i)_t y(i)_t \leq M(i)_t$$

for all i and t.

It follows that the demand for good i in period t is given by

$$U'(y_t) y_t^{\frac{\mu-1}{\mu}} y(i)_t^{\frac{1-\mu}{\mu}} = \alpha R(i)_t \tag{12}$$

where $R(i)_t$ is the gross nominal interest rate

$$R(i)_t = (1 + r_t)(1 + \pi(i)_t)$$

and $\pi(i)_t$ is the -currency i- inflation rate between period $t - 1$ and t. Furthermore, given that utility is linear in leisure, equilibrium real interest rates satisfy $r_t = \rho$

The demand for good i, can be written as

$$y(i)_t = y_t \left[\frac{R(i)_t}{R_t} \right]^{\frac{\mu}{1-\mu}} \tag{13}$$

where

$$R_t \equiv \left[\int_0^1 R(i)_t^{1/1-\mu} di \right]^{1-\mu}$$

As long as the cash-in-advance constraints are binding, (13) results in a demand for currency i

$$m(i)_t = m_t \left[\frac{R_t}{R(i)_t} \right]^{\frac{\mu}{\mu-1}} \tag{14}$$

where $m_t = \left[\int_0^1 (m(i)_t^{1/\mu} \right]^{\mu} di$.

The issuer of currency i faces an intertemporal budget constraint given by

$$\frac{M(i)_{t+1}}{P(i)_t} + d(i)_{t+1} = \frac{M(i)_t}{P(i)_t} + d(i)_t(1+\rho) + \Pi(i)_t$$

where $d(i)_t$ is the debt of the i-currency issuer at time t, in units of the consumption good, and $\Pi(i)_t$ are the profits of the money issuer in units of the consumption good. It also faces the corresponding non-Ponzi constraints guaranteeing that the present value budget constraint is well defined. For simplicity we will assume that $d(i)_0 = 0$ for all i. The present value of profits are

$$\sum_{t=0}^{\infty} \beta^t \Pi(i)_t = \sum_{t=1}^{\infty} \beta^t \left((R(i)_t - 1)\, m(i)_t \right) - \frac{M(i)_0}{P(i)_0} \tag{15}$$

where $m(i)_t = \frac{M(i)_t}{P(i)_t}$.

In order to maximize the present value of profits, firms must choose $\pi(i)_t$ to maximize

$$(R(i)_t - 1)\, m(i)_t$$

taking $r_t = \rho$ and (14) as given. They must also minimize $\frac{M(i)_0}{P(i)_0}$. Notice that, as in standard (single currency) monetary models, a monetary policy for the i-currency issuer consists on a current price level and a sequence of future nominal interest rates: $\left(P(i)_0, \{R(i)_t\}_{t=1}^{\infty}\right)$.

3.1 Currency competition with full commitment

We now assume that currency issuers can fully commit to a monetary policy. That is, once they announce a policy, $(P(i)_0, \{R(i)_t\}_{t=1}^{\infty})$, they implement it. We now characterize the corresponding monetary equilibrium

Regarding monetary policies, optimality requires the initial price level to be arbitrarily high such that the real value of initial outstanding money holdings (liabilities) $\frac{M(i)_0}{p(i)_0}$ become zero. This is achieved through a big open market operation in which the currency is sold back to the consumers. Each currency issuer takes a negative position in bond holdings, in an amount equal to the real quantity of money. In subsequent periods, the currency issuer collects the real rate of interest on these bond holdings, as well as the inflation rate on real money holdings, corresponding to future money issuing..

To characterize the problem of maximizing time t profits, notice that to maximize

$$(R(i)_t - 1)\, m(i)_t$$

subject to (14), results in the choice

$$R(i)_t = \mu$$

This is not surprising, since this maximization problem is the same, in the monopolistic competition model, as that of maximizing (4) subject to (3). We only need to identify the gross nominal interest rate, in the currency competition model, $R(i)_t$ with the price $p(i)_t$ in the monopolistic competition model, and $m(i)_t$ with $y(i)_t$. As in the previous model, with these prices, the consumption of the goods, $\overline{y} = \overline{m}$, is constant and satisfies $U'(\overline{y}) = \alpha\mu$.

It follows that, as currency substitution increases, i.e., $\mu \searrow 1$, nominal interest rates tend to zero, i.e., $(R(i)_t - 1) \searrow 0$, which is supported by a deflationary monetary policy, i.e., $\pi(i)_t \searrow (\beta - 1)$. In other words, with perfect substitution of private currencies the monetary equilibrium is efficient and the Friedman rule is implemented. In summary, with full

commitment, Hayek's conjecture, that efficient monetary equilibria can be achieved through currency competition, is satisfied.

Nevertheless, as in standard (single currency) monetary models, the full commitment monetary policy is time inconsistent. This can easily be seen by considering how the budget constraints of a currency issuer evolves over time. At time t, the budget constraint is

$$\sum_{j=t}^{\infty} \beta^{j-t} \Pi(i)_j = \sum_{j=t+1}^{\infty} \beta^{j-t} \left((R(i)_j - 1) m(i)_j \right) - \frac{M(i)_t}{P(i)_t} - \frac{d(i)_t}{\beta} \qquad (16)$$

Thus, if given the option to change plans at time t,which we rule out when assuming full commitment, the currency issuer will find it optimal to expand the money supply and let $P(i)_t$ increase without bound. The reason is that the real money demand is decreasing in the nominal interest rates, i.e., in expected future prices. However, once consumers have made their currency decisions, they are stuck with the outstanding money holdings and the nominal money demand is rigid with respect to the realized price. We turn now to analyze the case that full commitment can not be granted.

3.2 Currency competition without full commitment

As there is a parallel between monopolistic competition with perfect observability and currency competition with full commitment, there is a parallel between monopolistic competition with unobservable quality and currency competition without full commitment. More specifically, in both models firms compete is in an announced price, in the monopolistic competition model it is the price of the good or service, in the currency competition model is the nominal interest rate, or the inflation rate. With perfect observability in the first model and with full commitment in the second, realized prices must coincide with the announced prices. With unobservable quality in the first model and lack of commitment in the second, realized prices may differ from the announced ones. In fact, in such a case, firms maximize short run profits by setting an

arbitrarily large realized price, which in the quality model corresponds to choosing low quality and in the currency model to inflate away current money holdings (i.e., in making "the quality of outstanding money" arbitrarily low). In both models, the timing is very important[11]: consumers purchase services before they observe the quality they yield, in one, and they purchase monies before they observe the return they yield, in the other; in both models, consumers must form their expectations on realized prices, based on past information and current announcements, and, in both models, reputation is what may prevent firms from "flying-by-night."

More formally, while monopolistic firms sequentially choose $\left(\widetilde{p(i)}_t, q(i)_t\right) \equiv \left(\widetilde{p(i)}_t, \frac{\widetilde{p(i)_t}}{p(i)_t}\right)$, currency issuers sequentially choose $\left(\widetilde{R(i)}_{t+1}, \frac{\widetilde{R(i)_t}}{R(i)_t}\right)$, where, in period t, $\widetilde{R(i)}_{t+1}$is the announced gross nominal interest rate and $R(i)_t$ the realized, or *ex-post*, rate. Given that $R(i)_t = (1+\rho)(1+\pi(i)_t)$, we can equivalently say that currency issuers choose $\left((1+\widetilde{\pi(i)}_{t+1}), \frac{(1+\widetilde{\pi(i)_t})}{(1+\pi(i)_t)}\right)$. In fact, given the last period announcement $(1+\widetilde{\pi(i)}_t) = \frac{\widetilde{P(i)_t}}{P(i)_{t-1}}$ and the -within the period- observed $P(i)_t$, we can say that currency issuers choose $\left(\widetilde{R(i)}_{t+1}, \frac{1}{P(i)_t}\right)$. The difference of timing between the two models corresponds to the fact that in the first model competition is on announced current -period t- prices, while in the second on announced -between period t and $t+1$- interest rates.

We can now define a "*Sequential Currency Competition Equilibrium*" (SCCE) in a similar fashion as we have defined SMCE in the monopolistic competition model. Histories are given by $h_0 = \{P(i)_0, \text{ for all } i\}$ and, for $t > 1$, $h_t = \{h_{t-1}, R(i)_t, \text{ for all } i\}$; i-currency issuer strategy is given by $\sigma^f_{i,t}(h_t) = \left(\widetilde{R(i)}_{t+1}, \frac{1}{P(i)_t}\right)$; consumers make their decisions $\sigma^c_t(h_t, \widetilde{R(i)}_{t+1}) = \{n_t, y(i)_t, M(i)_{t+1}, \text{for all } i,\ b_{t+1}\}$ based on their beliefs $\upsilon^{c,i}_t(h_t, \widetilde{R(i)}_{t+1})$, which denote the assessed probability that, given history, h_t,realized nominal interest rates are as announced, i.e., $R(i)_{t+1} =$

[11]In a paper that also addresses the issue of competition in a time inconsistency setting, Kehoe(1989) used a different timing and that is why he obtains the result that Bertrand competition does lead to the efficient outcome.

$\widetilde{R(i)}_{t+1}$. A SCCE is an *assessment* $\left((\sigma^c, \upsilon^c), (\sigma_i^f)\right)$ which is *consistent* and, such that, for every (t, h_t), $\sigma_{i,t}^f(h_t)$ solves the maximization problem of the i-currency issuer; $\sigma_t^c(h_t, \widetilde{R(i)}_{t+1})$ solves the consumer's problem given his beliefs $\upsilon_t^{c,i}(h_t, \widetilde{R(i)}_{t+1})$, and all markets clear, including the perfectly competitive goods markets. As with SMCE, there is a worst SCCE in which currencies are not held, since agents expect realized nominal interest rates to be arbitrarily large.

To see more explicitly how currency competition and reputation interact, consider the problem of whether a stationary gross nominal interest rate, $R(i)$, is sustainable as a SCCE. Suppose that the i-currency issuer considers a deviation in period t, from the announced $\widetilde{R(i)}_t$ by printing arbitrarily large amounts of money, i.e., sets $\frac{1}{P(i)_t} = 0$. As the issuer deviates, the agents expectations become $\upsilon_{t+s}^{c,i}(h_{t+s}, \widetilde{R(i)}_{t+1+s}) = 0$, $s \geq 0$. Thus, the demand for currency i becomes zero from time t on, i.e., $m(i)_{t+s} = 0$, $s \geq 1$, which means that the newly issued pieces of paper are worthless. In fact, the value of the outcome after the deviation is zero, but for the value of the outstanding real debt. The reason is that the deviation triggers a currency collapse for that currency, starting tomorrow. But, contrary to the monopolistic competition model with unobserved quality, the demand for money, being an asset, depends on future prices. Thus, the expectations of the currency collapse make the newly injected money be worthless today. Therefore, the present value of the benefits following a deviation are obtained by replacing the real value of money stocks from time t on by zeroes in the expression for profits (**??**)

$$V^D(i)_t = -\frac{d(i)_t}{\beta}$$

On the other hand, if the issuer does not deviate, the present value of the profits are

$$\begin{aligned} V^C(i)_t &= \beta\frac{(R(i)-1)m(i)}{1-\beta} - \frac{M(i)_t}{p(i)_t} - \frac{d(i)_t}{\beta} \\ &= \rho^{-1}(R(i)-1)m(i) - m(i) - d(i)_t\beta^{-1} \end{aligned}$$

The last equality follows from the fact that, in equilibrium, $m(i) = \frac{M(i)_t}{p(i)_t}$. It follows that, the i-currency issuer will chose not to deviate from the stated policy when

$$\left[\rho^{-1}(R(i)-1)-1\right] \geq 0$$

$$\text{i.e.,} R(i) \geq 1+\rho$$

or, equivalently, whenever $\pi(i) \geq 0$. Notice that, under such stationary profits are given by $\Pi(i)_t = (R(i)-1)m(i)$, for $t \geq 1$, while, by the present value constraint (15) $\Pi(i)_0 = -\frac{M(i)_0}{P(i)_0}$. That is, given initial money holdings, $M(i)_0$, profit maximization requires $\frac{1}{P(i)_0} = 0$. As in the previous model, the set of stationary SCCE is fairly large, although, given a stationary monetary policy $\left(\frac{1}{P(i)_0}, \pi(i)\right)$, i.e., $R(i)_t = (1+\rho)(1+\pi(i))$, $t > 0$, the SCCE allocation is uniquely defined. More formally, the following proposition parallels Proposition 3,

Proposition 5 *For any $\pi(i) \geq 0$, the policy $(0, \pi(i))$ can be supported as strategy of a stationary symmetric SCCE.*

The reason why zero inflation is important is because of the timing of collection of revenues for the issuers. Remember that along the commitment solution, the issuers make initial money holdings be valueless and, by an open market operation they sell back the new money balances to the consumers. Thus, at the first period the issuers hold positive assets in an amount equal to the real balances. From those assets they collect the real rate of interest, ρ. Thereafter, they also collect the inflation rate times the real money balances every period. If they deviate, they will keep the real asset holdings only[12]. Thus, as long as the returns they make with the inflation tax are non-negative, they have no incentives to deviate.

[12]Note that if the issuer were forced to hold their own currency denominated assets, then the efficient outcome could be supported as a SCCE.

3.2.1 Regular beliefs

As in Section 2, we can restrict the set of SCCE, and allow for competition to play its role, by assuming that beliefs satisfy the regularity conditions. *Weak monotonicity* and $\varepsilon-$ *positive beliefs* can be similarly defined by replacing $\underline{p(i)}$ by $\widetilde{R(i)}$ and $p(i)/1$ by $R(i) = \widetilde{R(i)}$ (resp. $p(i) \neq \widetilde{p(i)}$ by $R(i) \neq \widetilde{R(i)}$). Then, the parallel of Proposition 4 is

Proposition 6 *There is a unique stationary* Regular Currency Competition Equilibrium *(RCCE), which is characterized by the inflation rate* $\pi(i) = \max\{\mu - (1+\rho), 0\}$ *and the initial price* $1/P(i)_0 = 0$.

It follows that, without full commitment, when privately issued currencies are very close substitutes inflation is zero and the nominal interest rate ρ. That is, the monetary equilibrium is not efficient and the Friedman rule is not implemented. In summary, without full commitment, Hayek's conjecture, that efficient monetary equilibria can be achieved through currency competition, is not satisfied.

4 Conclusions

In this paper we have tried to clarify three related -but distinct- issues. First, how competition and reputation interact when, on the one hand, firms are subject to competition, but, on the other hand, such competitive pressure does apply to all their decisions. These decisions, nevertheless, may be subject to reputational pressures. Second, we have seen how in this general context two -apparently, very different- economic problems share the same basic features: monopolistic competition with experienced goods and currency competition. Third, we have seen that sequential (or sustainable) equilibria may not have much predictable power, and misrepresent the role of competition, in reputational models, but that imposing a weak regularity condition on beliefs results in equilibria, where competition plays a crucial role in enhancing efficiency. Nevertheless,

it is within the nature of the reputational mechanism that competitive pressures can not achieve full efficiency. A particular corollary of these results is that Hayek conjecture, that efficient monetary equilibria can be achieved through currency competition, is not satisfied if currency suppliers make sequential decisions. Any of these three issues explored here suggest further work. In particular, alternative forms to model currency competition may be more suitable to relate our results with historical experiences. We leave this for future research.

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